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Analysis and Optimization of Wind Turbine Noise under Uncertainty

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Abstract

In an era of a great widespread of wind turbine power generation, designs are needed which are both efficient and minimally disruptive to surrounding communities by severe limitations given by rules. An imperfect manufacturing process, insect contamination, variability of methodological conditions could lead wind turbines in operating not exactly at their design conditions. The results of these uncertainties might lead to unexpected lower performances of the aeronautic design as well as variability of their predicted noise. Even a deterministically optimized design could exceed, under uncertainty, the limits given by laws and the performance degradation could result in a smaller amount of energy than forecasts. In this work a model of wind turbine is presented, considering a variable pitch and rotational speed control, fluid-structural interaction and acoustics. Hence a typical turbine design is analyzed under uncertainty by the use of a Simplex Elements Stochastic Collocation (SESC) method. The presented non–intrusive Uncertainty Quantification (UQ) method is based on adaptive grid refinement of a simplex elements discretization in probability space. The approach is equally robust as Monte Carlo (MC) simulation in terms of the Extremum Diminishing (ED) robustness concept. Six different forms of aerodynamically produced noise will be superimposed to calculate the total aeroacoustic signature of an operating wind turbine. A process of design optimization via genetic algorithms will be explored for the reduction of noise when uncertainties are not neglected.

Introduction

Wind turbine reliability plays a critical role in the long-term evolution of wind-based energy generation. The computational assessment of failure probability or life expectancy of turbine components is fundamentally hindered by the presence of large uncertainties in both the environmental conditions and blade geometry and structure. Rigorous quantification of the impact of such uncertainties can fundamentally improve the state-of-the-art in computational predictions and, as a result, provide aid in the design of more cost-effective devices.
In the following we will describe a computational framework constructed around tools developed mainly at the National Renewable Energy Laboratory (NREL). These tools are essentially deterministic: once the wind-turbine configuration and other input conditions are specified, the solution is uniquely determined without vagueness. On

the other hand, when uncertainties are present, the results have to be expressed in a non-deterministic fashion either probabilistically or as ranges of possible outcomes. In this work we focus on the former, and describe the uncertainties as random variables. At this point the computations become probabilistic in nature and it is necessary to propagate the input variability into the output of interest (quantity of interest, QoI). The approach we follow here is strictly non-intrusive, in the sense that the existing tools are used without modifications, but the solution - or more precisely, their probability distributions - are constructed performing an ensemble of deterministic analysis.

A multi-physics low-order model for wind turbines

Wind turbines are multi-physics devices in which the aerodynamic performance, the structural integrity of the blades, the energy conversion toolbox and the acoustic impact have to be carefully examined to achieve an effective design. Each one of these aspects introduces considerable hurdles for detailed simulations. The aerodynamic performance is dominated by the design of the blade cross-sections. The sections are typically laminar-flow airfoils to reduce the overall drag. The flow characterization is complicated by the need to predict laminar/turbulent transition under a variety of clean and perturbed wind conditions, the inherent angle of attack variability associated to rotation, the presence of dynamic stall, aeroelasticity, etc. In spite of the development of advanced computational fluid dynamic tools that can predict with reasonable accuracy the aerodynamic performance of rotors,1 the computations remain extremely expensive and often rely on simple models to capture important effects, such as transition, and are generally not considered to be predictive for extreme events such as stall. In this work, we focus on building a flexible computational infrastructure based on low-fidelity models that are connected together in a matlab environment called EOLO[36]. There are two main advantages resulting from this choice: i) control and flexibility in using different models developed
for capturing complex phenomena, ii) low computational cost. It is the second aspect that fundamentally enables us to perform analysis under uncertainty. In the following we introduce the various computational tools that are used to perform the deterministic analysis. The uncertainty quantification methodologies are described in the next section.

The geometrical description of the turbine blades is based on the specification of three airfoils at the root, mid-span and tip. Simple linear interpolation is used to construct the geometry at the other cross-sections and the local aerodynamic (e.g. two-dimensional) analysis is carried out using a potential flow method with interactive viscous correction. The tool we used is Xfoil[9] which includes a model for boundary layer transition based on the e^N method. Xfoil is used to determine the aerodynamic force coefficients polars in a range of angle of attacks from -15° to 25° to cover the range of incident angles experienced during a full rotation. Xfoil is not expected to be accurate in the prediction of stall, because of the presence of extensive flow separation and possibly unsteady effects. Hence a correction to the polar curve is introduced, based on Viterna[10] and Corrigan models which provide a correction of

the lift and drag coefficient at high angle of attack. A final correction to the aerodynamic coefficients is employed due to the presence of finite-span effects.

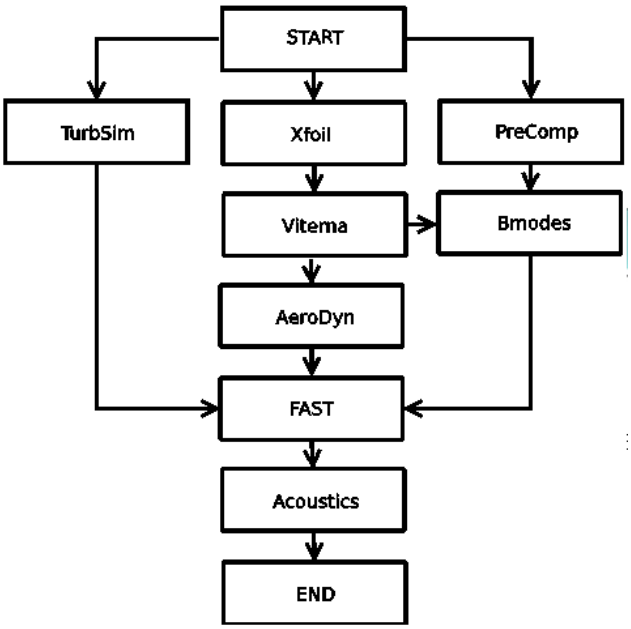


Figure 1 - Multi-physics computational framework to perform analysis of wind turbine: EOLO flowchart

Fluid structure interactions play an important role in the determination of the structural integrity of the turbine blades and in the overall aerodynamic performance. The geometrical description of the blade is used as a starting point to define span-varying properties relevant to its composite structure. The NREL PreComp[2] computes cross-coupled stiffness, inertia and offsets of the blade shear center, tension center, and center of mass with respect to the blade pitch axis. These quantities are then used to determine a low-order model for the rotor, tower and drivetrain shaft. Specifically, the characteristics of a rotating-beam equivalent to the blade are computed using NREL BModes[5] a finite element code that evaluates the deformation modes.

The Viterna corrected polars at certain nodes along the span, the flapwise and edgewise Bmode smodal shapes and the PreComp properties are then used as input to NREL FAST[3] (Fatigue, Aerodynamics, Structures, and Turbulence) which is a comprehensive aeroelastic simulator capable of predicting both the extreme and fatigue loads of two- and three-bladed horizontal-axis wind turbines. This code is based on the NREL AeroDyn[4] solver, an element-level wind-turbine aerodynamic analysis routine. It requires information on the status of a wind turbine from the dynamics analysis routine and a wind file describing the atmospheric conditions. It returns the aerodynamic loads for each blade element to the dynamics routines.

The aerodynamic performance of wind turbines is dominated by the wind conditions. Atmospheric boundary layers are subject to large variability in wind direction and intensity with largely unsteady dynamics and frequent gusts. In EOLO we generate realistic wind conditions using the NREL TurbSim[6] tool, which constructs a stochastic inflow with a precisely specific velocity fluctuation spectrum.

The NREL prediction of aeroacoustic noise is based on six different noise sources (Fig. 2) that are assumed to independently generate their own noise signature. The assumption of independence is based on the idea that the mechanisms for each noise source(namely turbulent boundary layer trailing edge, separating flow, laminar boundary layer vortex shedding, trailing edge bluntness vortex shedding, and tip vortex formation[24,26]) are fundamentally different from each other or occur in different locations along a turbine blade, such that they do not interfere with one another.

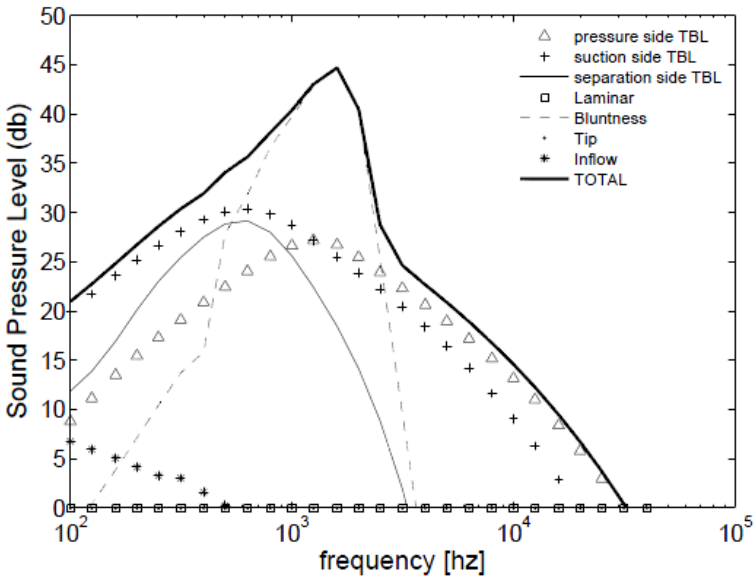


Figure 2. Breakdown of the noise generated by a 50kw wind turbine at a microphone located at (x,y,z)=(-20m,0m,0m)

The various tools briefly described in the previous subsections are glued together in a multi-physics simulation process using matlab. The overall driver script, EOLO handles the transfer of information between the various tools and then collects the final outputs and computes statistics. A flowchart of the process is reported in Fig. 1; it is clear that modifications to the framework can be handled in a simple way, for example substituting the aerodynamic performance evaluation module (Xfoil and Viterna) with a computational fluid dynamic solver. EOLO also provides a unique interface for the entire process (from inputs to outputs) that is directly connected to the uncertainty quantification tools presented in the next section.

Uncertainty Quantification

The simulation environment described above can be effectively used to study wind turbine performance in the absence of uncertainties. In this section we introduce a methodology that enables us to characterize the effect of variability in wind conditions, manufacturing tolerance and insect contamination. As mentioned earlier we limit our analysis to uncertainties that can be described using random variables (aleatory uncertainties) and, therefore, our goal is to construct a probabilistic framework around the EOLO environment. The most straightforward choice is to perform Monte Carlo (MC) sampling in which many deterministic

simulations corresponding to randomly chosen wind conditions for example, are performed and a statistical characterization is obtained directly from this ensemble. It is well known that typically a very large ensemble is required to achieve convergence of the QoI statistics. Due to the relatively slow convergence rate of Monte Carlo simulations, other uncertainty quantification methods have been developed based on a polynomial approximation of the response. Stochastic Collocation (SC) is a widely used example of such a method, which is based on sampling Gauss quadrature points and using Lagrangian polynomial interpolation in probability space. However, due to the structured grid of the quadrature points in multiple random dimensions, the spectral convergence of the Stochastic Collocation method reduces significantly with an increasing number of uncertainties. Here, the Simplex Stochastic Collocation (SSC) method[7,8] is presented that combines the effectiveness of random sampling in higher dimensions with the accuracy of polynomial interpolation. It also leads to the superlinear convergence behavior of Stochastic Collocation methods and the robustness of Monte Carlo approaches. SSC is based on adaptive grid refinement of a simplex elements discretization in probability space. The approach is equally robust as Monte Carlo (MC) simulation in terms of the Extremum Diminishing (ED) robustness concept. The initial samples are located at the parameter range extrema and one at the nominal conditions, see Figure 3a for a two-dimensional example. The discretization is adaptively refined by calculating a refinement measure based on a local error estimate in each of the simplex elements. A new sampling point is then added randomly in the simplex with the highest measure and the Delaunay triangulation is updated. The sample is confined to a sub-domain of the simplex to ensure a good spread of the sampling points, see Figure 3a. The sampling procedure is stopped when a global error estimate reaches an accuracy threshold. In the wind turbine simulations and other large-scale problems, it is possible that one of the deterministic computations for a specific sample of the random parameters does not converge or gives an unrealistic result. For the Stochastic Collocation method such a failure of one of the quadrature samples would be a serious problem in computing statistical moments.

(a) Refinement of the initial mesh (b) Refined mesh for 17 samples

Figure 3. Simplex Stochastic Collocation discretization of a two–dimensional probability space.

the randomized sampling. It is handled by introducing a check of the correct execution of the samples into the algorithm. If an unconverged sample is detected, then the failed sample computation is automatically restarted for another randomly sampled point in the refined simplex element. In the analysis performed in this paper,

this has proven to be an effective approach for dealing with erroneous samples, which shows the flexibility of the SSC method in complex computational problems.

Deterministic Optimization

Genetic Algorithms (or GAs) are adaptive heuristic search algorithms based on Darwinian natural evolution processes. In analogy to living organisms in nature, individuals of a population can be managed by computers as a digital - binary or floating point- DNA with the diversity associated to design variables optimization problem. A genetic algorithm consists of a finite population of individuals of assigned size, each of them usually encoded as a string of bits named genotype, an adaptive function, called fitness, which provides a measure of the individual to adapt to the environment, that is an estimate of the goodness of the solution and an indication on the individuals most likely to reproduce, semi-random genetic operators such as selection, crossover and mutation that operate on the genotype expression of individuals, changing their associated fitness. In this application, we follow Zhong and Qiao's work using the B-spline method to parameterize the geometry. The shape modifications are parameterized by fifth order B-splines with a nominal uniform knot set.

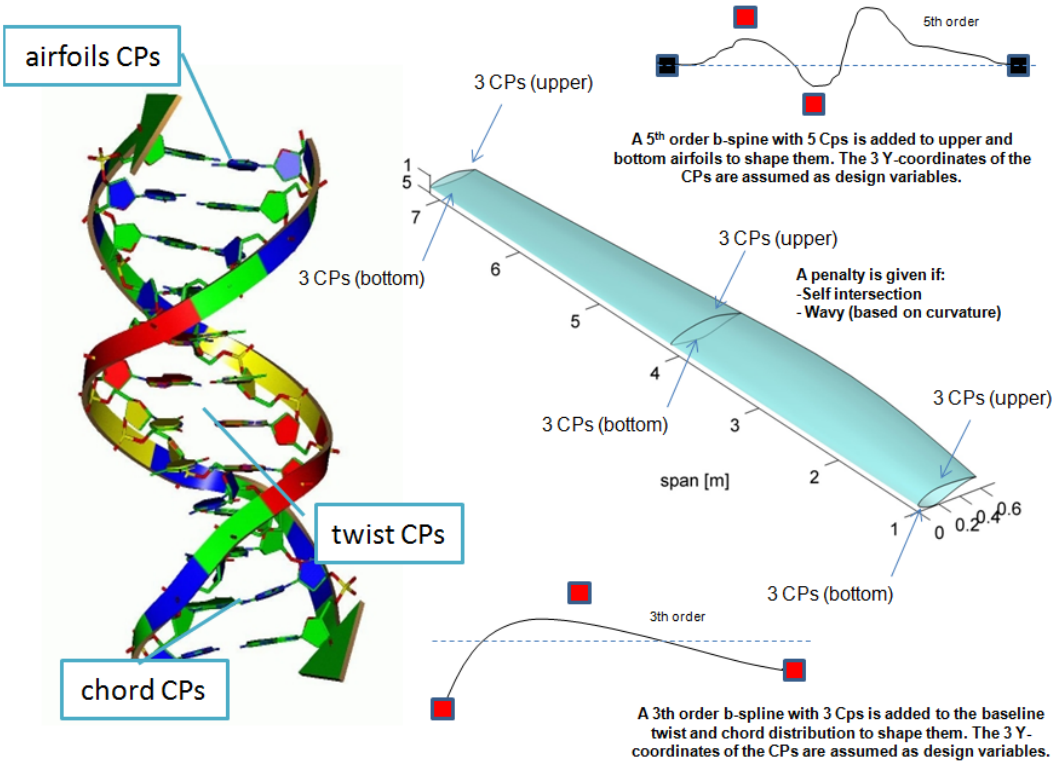


Figure 4. Determinist optimization, parameters definition

Data Assimilation

The energy produced by a wind turbine is usually expressed as an annual average. Since production falls off dramatically as the wind speed drops, most of the time the wind turbine is producing well below its expected rate.[16] It is important to characterize the wind turbine behavior resulting from the measured wind variability to assess the effective performance. For land based turbines, the wind speed

distribution is usually approximated by a Weibull fit. [11] As an example, Downey[15] extracted data from the database http://winddata.com of eight sites that have wind speed measurements above 60 m in height. Following the same approach we extracted nominal wind speed, turbulence intensity and direction data at a site (Acqua Spruzza, Italy) where a wind turbine farm was built by ENEL S.p.A. to evaluate the performance of commercial medium-sized turbines operating in complex terrain and very hostile climate. A large collection of wind measurements is summarized in Fig. 5 in terms of wind speed and direction and turbulence intensity. The histograms of these three random variables are used directly as input for the uncertainty propagation methods described previously, after being converted into continuous probability density functions (for each of the input variables) via linear interpolation. Note that no information regarding the correlation of the three random variables is available, and therefore we assume that the inputs are independent. The wind data readily available provide an estimate of the wind speed at a certain height. To construct the wind conditions at the actual rotor hub height (24 meters) we use a classical[13] scaling law.

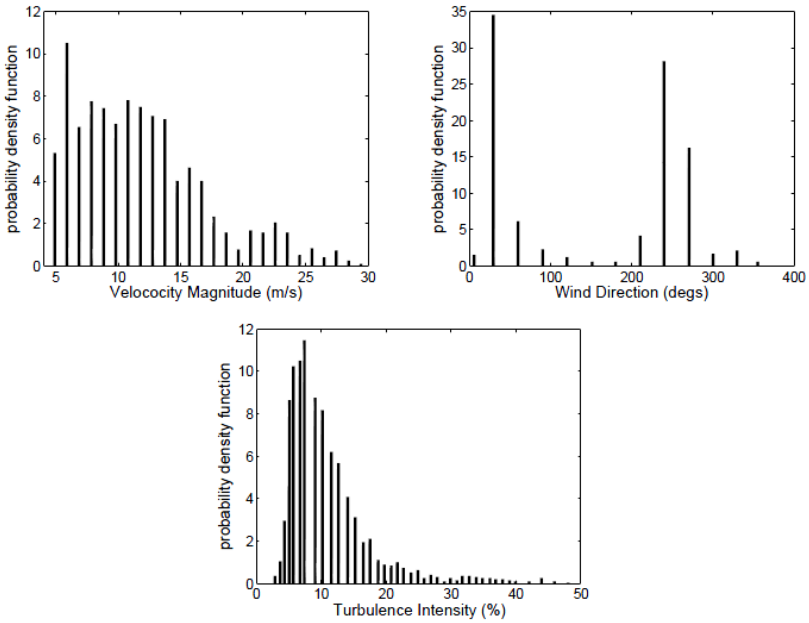


Figure 5. Wind speed, direction and turbulence intensity at the Acqua Spruzza, Italy site. The data is reported in terms of empirical probability distributions scaled from 40 to 24 meters.

Several studies on wind turbines[17–20] and fixed wings[21, 22] illustrate the effect of insect and dirt contamination on the overall aerodynamic performance. Insects are present in the lower layer of the atmosphere, with a density rapidly decreasing from ground level to 500 ft. Hardy and Milnecite[23]found that the morphology of insects is a function of the altitude and that estimation of the actual contamination depends on the operating conditions. In wind-turbines the effect of contamination can be particularly strong when the blade cross-sections are designed to support mostly laminar flows. The presence of insect contamination produces boundary layer disturbances that can lead to early transition to turbulence with a deterioration of the aerodynamic performance. This is the motivation for including insect contamination as a leading cause of uncertainty in the analysis of wind turbines. Crouch et al[25]studied experimentally the effects of surface protrusions (steps) on the transition

to turbulence in boundary layers. They also modified the e^N method to capture the observed transition modifications, via a reduction of the critical N-factor:

$$N_{crit} = N_{crit}{}^{0} - \Delta N_{crit}\left(\frac{h}{\delta^*}\right)$$

where h is the height of the step (i.e. the accumulated insect height)[m], δ^* is the boundary layer displacement thickness at the step location [m], N_{crit} accounts for the local change in the stability characteristics at the step[-] and $N_{crit}{}^{0}$ is the clean value of the critical n-factor[-]. In this work we assume that the insect impact produces a roughness that leads to a possible modification of the N-factor. We consider three independent variables describing the N-factor ranging from clean conditions (N_{crit}= 9) to transition bypass (N_{crit} = 1) at the root, mid-span and tip sections.

There is a general agreement that airfoil shape, twist and chord length imperfections are detrimental to aerodynamic performance, but only limited quantitative data is available in the open literature about their origin and quantitative effects. Loeven and Bijl[30] used a Polynomial Chaos Framework for the quantification of airfoil geometrical uncertainties. Ilinca, Hay, and Pelletier[27] treat shape sensitivities of unsteady laminar flow around a cylinder in ground proximity. Etienneet al [28] investigated shape sensitivities of flexible plates in a flow domain. Gumber, Newman, and Hou[29] included first order moments in robust design optimization of a 3D flexible wing with uncertain wing geometry. The geometry of a manufactured wind turbine airfoil is generally different from the nominal design mainly because of manufacturing tolerances. It is generally difficult to characterize probabilistically the effect of these tolerances; in this work we focus on errors associated with the protrusion process, where the blade is constructed as a sequence of cross-section. We assume that the twist of the blade (the section orientation with respect to a nominal plane) is imprecise. As before, we assume that we can describe the uncertainty using three independent parameters (with uniform probability distributions ranging from -2° to 2°) associated to the twist at the root, the mid-span and the tip section.

Uncertainty Quantification of a 50kw wind turbine

The AOC 15/50 is a downwind turbine, i.e. its blades rotate downwind of the drive train assembly. Furthermore, it has no active yaw control and depends on its blades to track the wind. This turbine is the evolution of the rugged and reliable Enertech E44, many of which were installed in the 1980's and are still running today. Independent analysis and testing at NREL, the Netherlands Energy Research Foundation (ECN), RISO Laboratory in Denmark, the Atlantic Wind Test Site (AWTS) on Prince Edward Island and other sites around the world verify that the AOC 15/50 wind turbine generators are very reliable in even the harshest weather conditions. The AOC 15/50 is designed for simplicity to minimize maintenance requirements and to be able to safely operate in normal and extreme conditions. The AOC 15/50 has been investigated using the uncertain meteorological conditions of Figure 5. In this case EOLO is driven by the SSC routines and the uncertainties are injected trough Turbsim; the statistics are constructed performing an ensemble of deterministic analysis. For reasons of economy, the wind history during the turbine's

approximately twenty year life is reduced to 10 minute periods (or load cases) at each wind speed. [16] The latitude chosen for the turbulence model is 41 degrees, matching the data extracted from the Acqua Spruzza site. The Von Karman spectral model for the meteorological boundary conditions has been chosen in this application, assuming neutral atmospheric condition. [33,34] The Monte Carlo samples on the response surface obtained by the simplex reconstruction are shown in Figure 6; the reader can notice that the samples follow the input distribution. A three color (red to blue) map has been introduced to relate the samples to the effective value of the power coefficient in the domain: the red points correspond to high power extracted by the wind turbine.

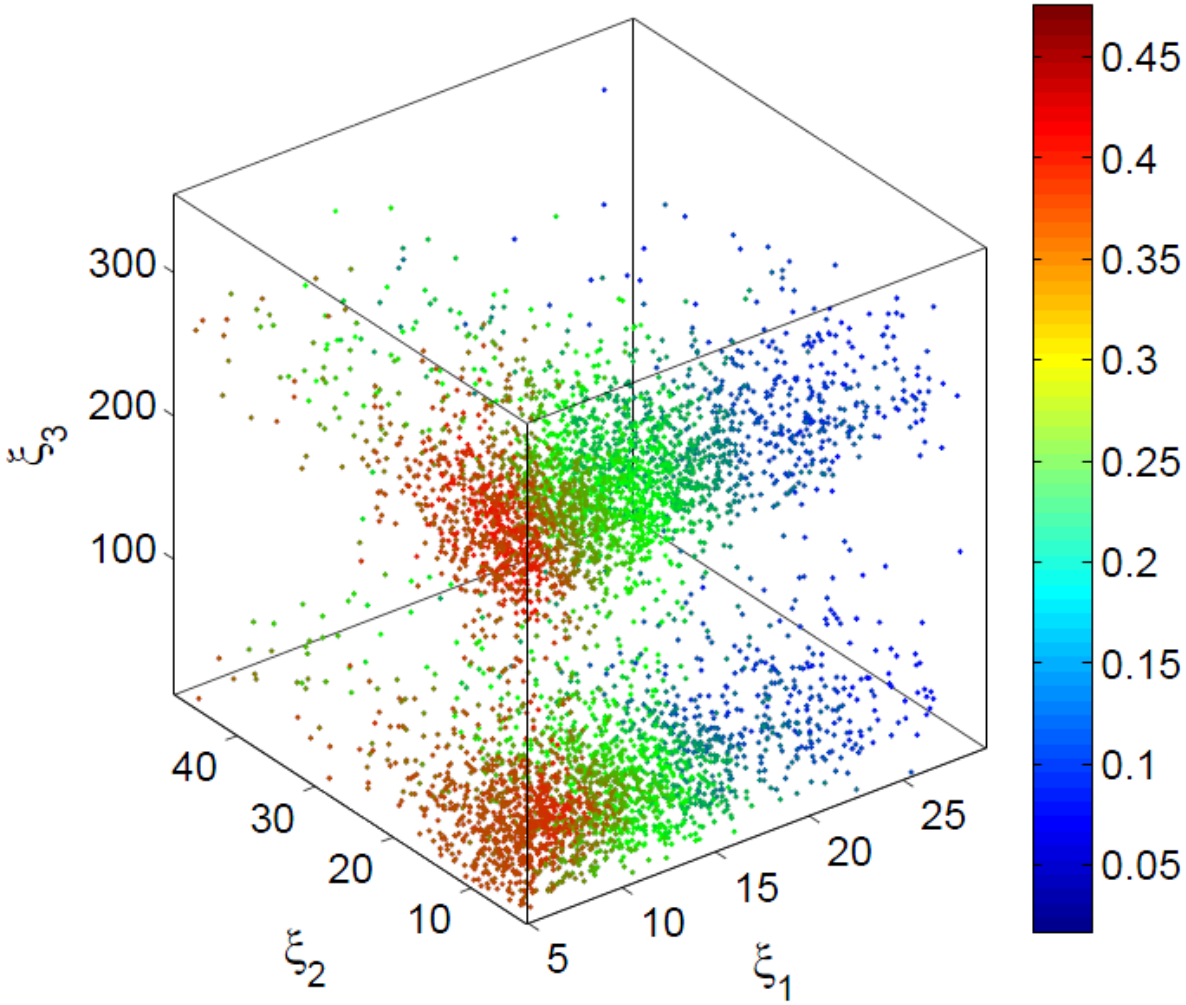


Figure 6. Monte Carlo samples for meteorological conditions.

The map reveals that improved efficiency is achieved for moderate wind speeds (5-12 m/s) and low turbulence levels (2-10 percent), while other conditions lead to decreased performance. The probability distributions fall completely below the deterministic characteristics of the wind turbine given by the vertical lines. The uncertain output for the power coefficient ranges from approximately 0 to the deterministic value of approximately 0.45. The sound pressure level varies uniformly between 34 to 45 dB. These results show that the realistic uncertainty in the wind

speed direction, and turbulence intensity has a large impact on the wind turbine performance.

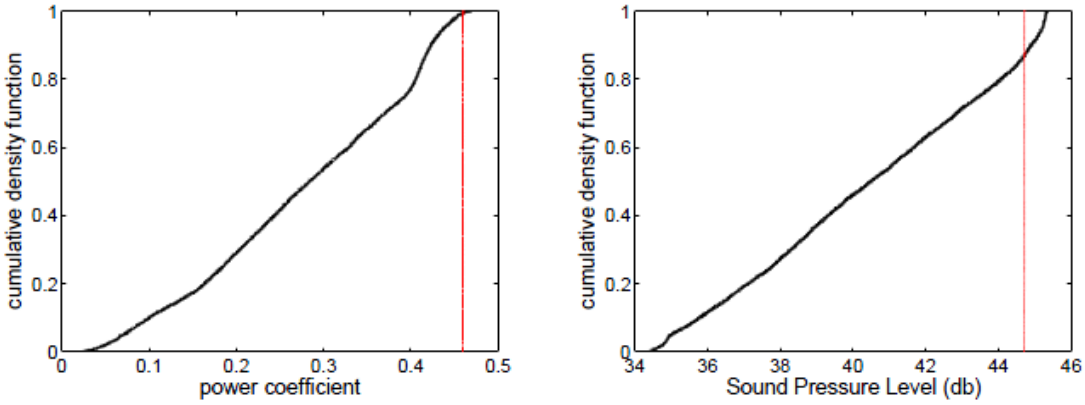


Figure 7. Meteorological conditions: cumulative density function of the power coefficient and Sound Pressure Level. The red lines represent the deterministic conditions.

The convergence of the mean and standard deviation of the sound pressure level is shown in Figures 8 up to 70 samples in the SSC discretization. The mean value of the output shows fast convergence in the first 20 samples to a value significantly lower than the deterministic value. Increasing the number of deterministic solves to 70 does not significantly change the mean value. This is confirmed by the decreasing error estimate intervals with an increasing number of samples. The higher moment of the standard deviation shows, as expected, a slower convergence up to 40 samples with a relatively larger error estimate margin.

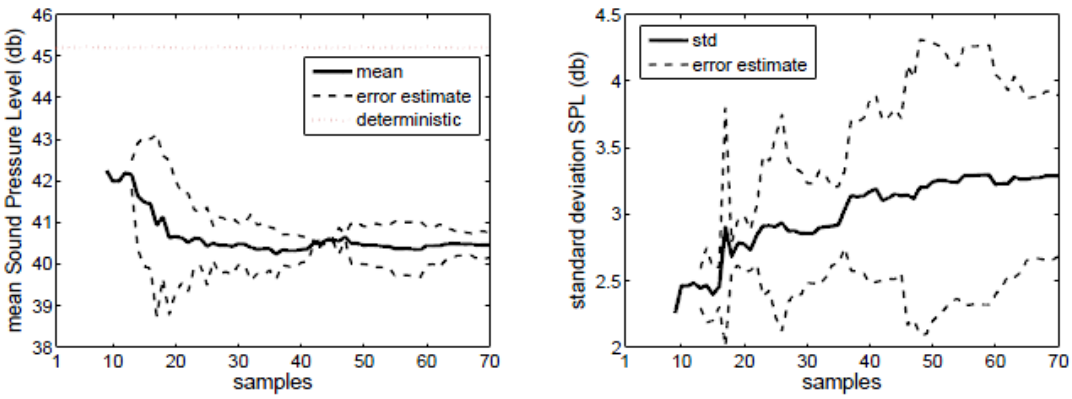


Figure 8. Meteorological conditions: convergence histories of the mean,variance and error of the Sound Pressure Level

Then the AOC 15/50 is investigated under insect contamination. In this case EOLO is driven by the SSC routines and the uncertainties are injected through the aerodynamic coefficients computed in Xfoil. This analysis illustrates a reduction of up to 16% in the power coefficient due to the insect contamination, while in the literature an effect of up to 50% has been reported. [17,18] This difference might be due to the present approach used to characterize the effect of the insect contamination. The variation of the perceived level of noise due to this source of uncertainty can be neglected.

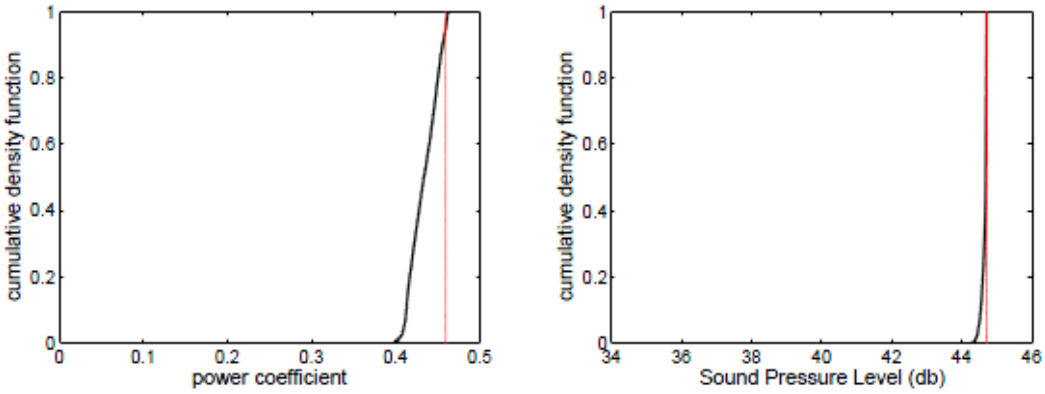


Figure 9. Meteorological conditions: convergence histories of the mean,variance and error of the Sound Pressure Level

The figure below shows the SSC convergence of the mean and the standard deviation of the output of interest. The error estimate is lower under uncertain meteorological conditions, therefore a smaller number of simplex points could have been used.

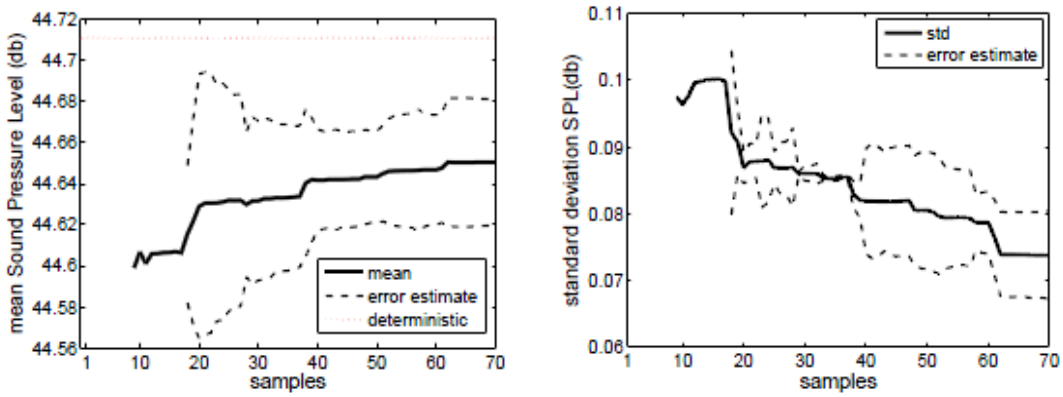


Figure 10. Insect contamination: convergence histories of the mean,variance and error of the Sound Pressure Level

In order to analyze manufacturing erros EOLO is driven by the SSC routines and the uncertainties are injected through the geometry pre-processor. In this framework we observed a reduction of the power coefficient by up to 7% with negligible change in the perceived noise.

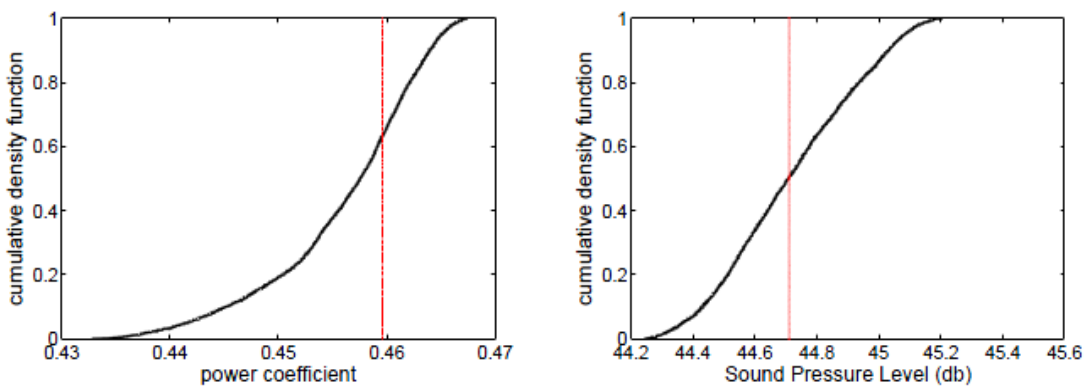
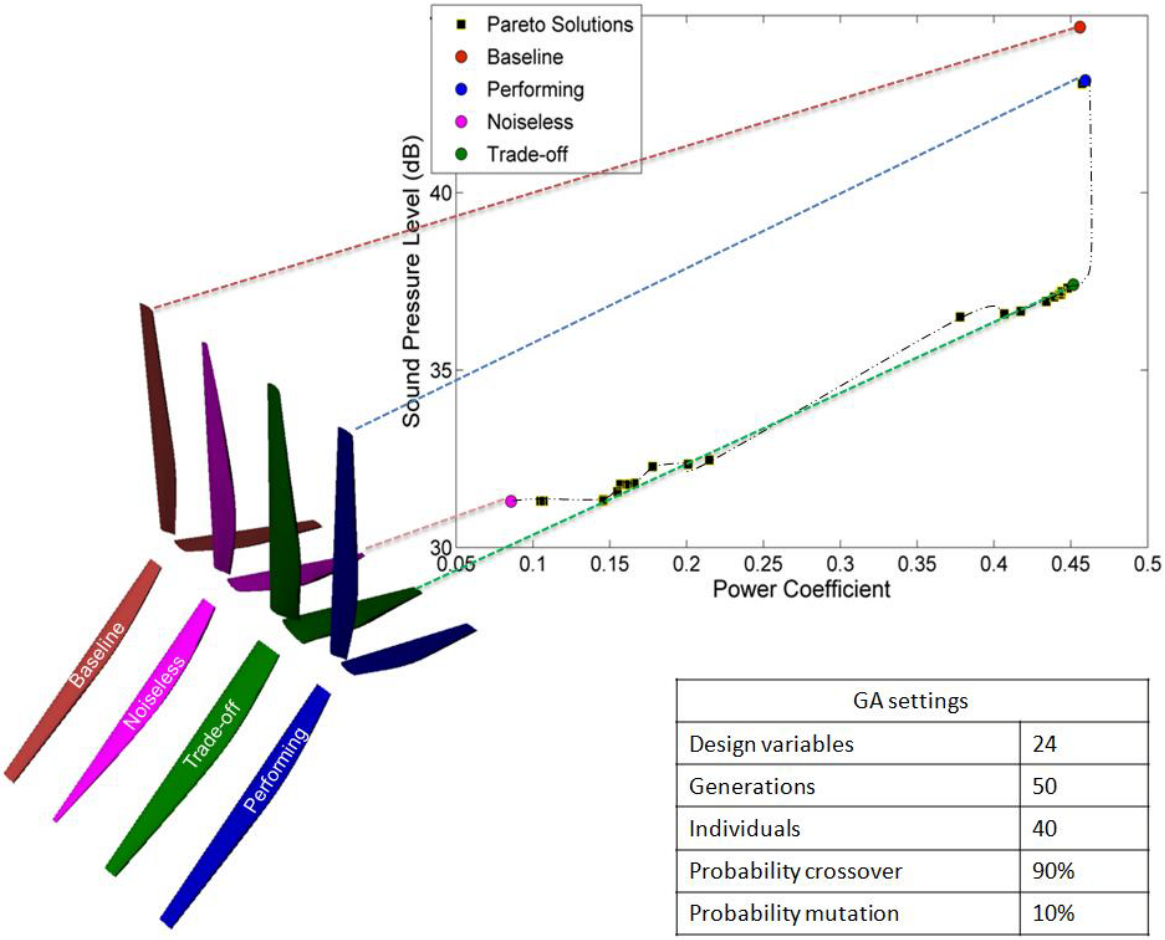


Figure 11. Manufacturing errors: cumulative density function of the power coefficient and Sound Pressure Level

Deterministic Optimization of a 50kw wind turbine

The objectives were to maximize the mean power coefficient [-] while reducing the Sound Pressure Level at a microphone located 20 m downwind the turbine at the ground level. The airfoils at the root, mid-span and tip of the blade where shaped adding to them 5th order B-splines on the bottom and upper surface, assuming the Y-coordinates of the control points as design variables. Additionally the initial twist and chord distribution were shaped adding to them 3rd order B-splines, assuming the Y-coordinates of the control points as additional design variables. Two checks were performed on each generated airfoil to verify if self-intersecting or wavy and in this case the candidate geometry was rejected. The baseline [red] case was already optimized by the producer but due to the sharpness of the Pareto front it was possible to find a trade off [green] design considerably less silent with a relative negligible reduction of the power coefficient.



GA settings	
Design variables	24
Generations	50
Individuals	40
Probability crossover	90%
Probability mutation	10%

Figure 12. Pareto front after 50 generations

Conclusions

The present study is a second step of a comprehensive analysis of wind turbine performance under uncertainty. We constructed a multi-physics low-order model EOLO that includes aerodynamic predictions, comprehensive structural

analysis and acoustic estimation. We identified three sources of uncertainty, namely wind condition, insect contamination and manufacturing tolerances, and successfully estimated their effect on aerodynamic performance and noise. Specifically, we demonstrate how the present uncertainties lead to a general decrease in performance with respect to the nominal (design) scenario. This penalization is also compounded with a likely variation in noise.

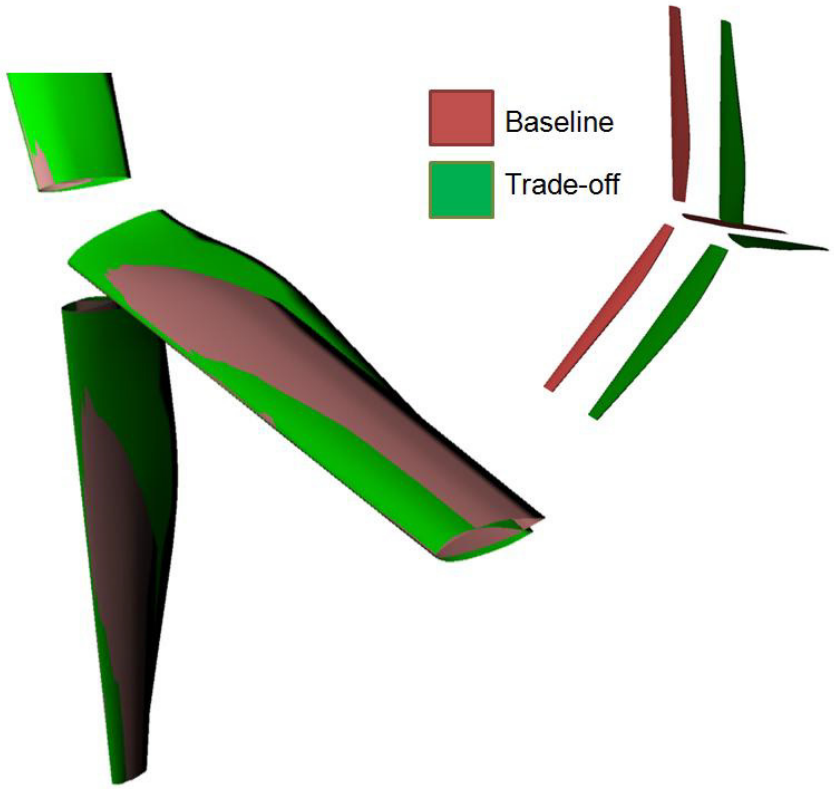


Figure 13. Trade-off design, a 3D detailed view

Additionally EOLO was used to optimize the shape of wind turbine blade proving that great achievements in terms on noise can be obtained without significant losses in the power coefficient. The next step of this research group is to optimize the shape of the blade taking in account uncertainties, combining the two applications of EOLO presented in this work in a nested loop using a novel probabilistically non-dominated sorting genetic algorithm.

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