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Noise from one stage of helical gears by wind turbine load

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Summary

The wind turbine gearbox has the most frequent failure problem and even a well-designed turbine can also produce the noise from gearbox. The conventional gearbox of the wind turbine uses planetary gears and one stage of external helical gears. This study deals with noise prediction from one stage of external helical gear system, which is a part of wind turbine gearbox. The system was modeled by shaft-bearing-circular plate with helical gears. Circular plate describes the gear housing. Excitations of the system are the aerodynamic torque. The aerodynamic torque was obtained from the rotational speed of the blade rotor, the average wind speed, the rotor swept area, the density of the air and the power coefficient for a fixed speed wind turbine. Mesh force due to excitation was calculated. Axial component of mesh force causes the longitudinal vibration of system and leads to transverse vibration of circular plate, which is acoustically significant. For longitudinal behavior of system, equation of motion of shaft-bearing-plate system due to axial excitation of motion was derived and dynamic analysis was performed. Plate displacement obtained by dynamic analysis becomes the source of noise. Noise was modeled by sound radiation from the clamped circular plate mounted in the infinite baffle. Noise characteristics of shaft-bearing-circular plate were investigated by numerical examples.

1. Introduction

Wind turbine noise has very important environmental concern. Many wind turbines use gear box type. However, even a well-designed wind turbine generates the noise from gearbox. Wind-turbine gear box has many diverse configurations to increase the speed. Conventional configuration of the gear box consists of planet gears and external helical gears. This work deals with noise prediction of a simplified gear system with one stage of external helical gears, which is a part of wind turbine gear box. Gear noise is initially caused by transmission error in gear meshing. The vibration from the gear meshing is transmitted to the gear housing through the shaft and the bearing. Housing vibration radiates the noise in air. Since the important contribution of gear noise is the structure-borne noise, the reliable and the simplified gear system modeling is required for the prediction of gear noise. Therefore, the purpose of this work is to develop the simplified gear system model for gear noise and to predict the noise from the simplified gear system with speed increasing helical gears.

For these purposes, the aerodynamic torque was obtained from the rotational speed of the blade rotor, the average wind speed, the rotor swept area, the density of the air and the power coefficient for a fixed speed wind turbine. The simplified gear system is modeled as a shaft-bearing-plate system. As an exciting force of the system, an axial shaft force from single degree of freedom model of speed increasing helical gears is calculated. Transfer matrix for the shaft-bearing-plate model is used to obtain the force and the displacement of the plate by the axial shaft force. The radiated noise from the damped circular plate mounted in the infinite baffle is calculated.

2. Mathematical Model

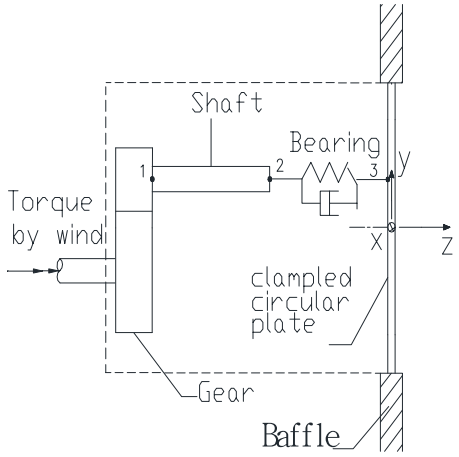

Fig.1 Model of wind turbine gearbox.

The model of the wind turbine gearbox consists of helical gears, a shaft, a bearing, and a clamped circular plate for a part of the gear housing as shown in Fig.1. Aerodynamic torque by wind is transmitted to helical gears. Axial excitation of helical gears is assumed to transmit to the shaft. Therefore, only longitudinal excitation of the shaft is considered. The shaft is modeled as a rod and the bearing as a parallel spring and dashpot. The gear housing is modeled as a damped thin circular plate connected with the shaft off the center of the plate.

The shaft and bearing modeling use a set of four pole parameters, which relate the input force and displacement to the output force and displacement. To do so, axial excitation of helical gears is first calculated. And the transfer matrix for shaft and bearing with four-pole parameters is used to obtain the force transmission from the shaft to the plate. Finally, the out-of-plane displacement for the thin circular plate with viscous damping and the sound pressure radiated from the plate due to gear axial excitation are calculated.

2.1 Aerodynamic torque by wind Aerodynamic torque for the wind turbine is caused by the interaction between the rotor and the wind. This section presents the aerodynamic torque for fixed speed wind turbines, which regardless of wind speed, turbine rotor speed is fixed and determined by the frequency of supply grid, the gear ratio, and the generator design. A wind speed is modelled as an average magnitude of the wind as follows:

$$V(t) = V_m \,. \tag{1}$$

The mechanical power extracted from the wind turbine is given by [1,2]

$$P = \frac{1}{2}\rho_A A_w C_p(\lambda) V(t)^3 \,. \tag{2}$$

If the fixed speed wind turbine is not pitch-controlled and assumes the constant pitch angle, the power coefficient for a fixed speed wind turbine, which means the fraction of the power in the wind extracted by the rotor, is given by [1,2]

$$C_p(\lambda) = 0.44\left(\frac{125}{\lambda_i} - 6.94\right)e^{-\frac{16.5}{\lambda_i}} \,, \tag{3}$$

where $\lambda_i = [\frac{1}{\lambda} + 0.002]^{-1}$, The tip speed ratio (TSR), which is defined as the ratio of the rotor-blade tip speed to the incoming free stream wind speed V(t), is given by[3]

$$\lambda = \frac{\Omega_m R}{V(t)} \,. \tag{4}$$

From the power of Eq. (2), the input torque applied on the low speed shaft produces as follows: [4]

$$T = \frac{\rho_A A_w C_p(\lambda) V(t)^3}{2\Omega_m} \,. \tag{5}$$

2.2 Gears Aerodynamic torque by wind produces the mesh force of helical gears. Helical gears have the mesh force in the axial, radial, and rotational direction. The axial force of helical gears is highly related to the rotational force and the important peaks of axial vibration correspond to those of rotational one. Therefore, this study calculates the force due to rotational vibration and it is transformed into axial forces by the base helix angle. The rotational

vibration by single degree of freedom is formulated as shown in Eq. (6) and it includes two forces resulting from aerodynamic torque and gear errors.

$$M\ddot{x} + 2\xi_G\sqrt{M\overline{K}_G}\,\dot{x} + K_G(t,x)x = F_G + F(t,x), \tag{6}$$

where $x = R_{b1}\theta_1 - R_{b2}\theta_2$, $F(t,x) = \sum_j K_{Gj}e_j$, $F_G = T/R_{b1}$.

This equation can be transformed into the normalized linear equation with non-dimensional variables as follows [5]:

$$\ddot{X} + 2\xi_G\omega_n\dot{X} + \omega_n{}^2 X = \omega_n{}^2 Y(t), \tag{7}$$

where $X = x/x_{st}, x_{st} = F_G/\overline{K}_G, \omega_n = \sqrt{\overline{K}_G/M}, Y(t) = y(t)/x_{st}$, $y(t) = \dfrac{F_G + F(t,x)}{K_G(t,x)}$.

The periodic forcing function Y (t) can be expressed in terms of harmonic functions as

$$Y(t) = Y_o + \sum_{i=1}^{\infty} Y_i \sin(i\omega t + \phi_i). \tag{8}$$

The steady state response is given by

$$X = Y_o + \frac{\sum_{i=1}^{\infty}\omega_n^2 Y_i \sin(i\omega t + \phi_i - \varphi_i)}{\sqrt{(\omega_n^2 - (i\omega)^2)^2 + (2\xi_G\omega_n i\omega)^2}}. \tag{9}$$

The response in the rotational direction is transmitted to the shaft by the damping and mesh stiffness force. Multiplying the shaft force in the rotational direction by the tangent of the base helix angle, the shaft force is transformed into the axial shaft force as follows [6]:

$$F_S = \frac{\sum_{i=1}^{\infty}\overline{K}_G Y_i\sqrt{(2\xi_G\omega_n i\omega)^2 + \omega_n^4}\sin(i\omega t + \phi_i - \varphi_{si})x_{st}\tan\beta_b}{\sqrt{(\omega_n^2 - (i\omega)^2)^2 + (2\xi_G\omega_n i\omega)^2}} + \overline{K}_G Y_o x_{st}\tan\beta_b. \tag{10}$$

2.3 Shaft-bearing-circular plate system Since the shaft and the bearing are modeled as a rod and the parallel spring and dashpot respectively, the four-pole parameters for the rod and the parallel spring and dashpot element are used[7]. The shaft and the bearing are connected in series. Therefore, the bearing force is calculated by multiplying the transfer matrix for the shaft and that for the bearing as follows:

$$\begin{Bmatrix} F_{3B} \\ U_{3B} \end{Bmatrix} = \begin{bmatrix} \alpha_{11}^2 & \alpha_{12}^2 \\ \alpha_{21}^2 & \alpha_{22}^2 \end{bmatrix}\begin{bmatrix} \alpha_{11}^1 & \alpha_{12}^1 \\ \alpha_{21}^1 & \alpha_{22}^1 \end{bmatrix}\begin{Bmatrix} F_1 \\ U_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}\begin{Bmatrix} F_1 \\ U_1 \end{Bmatrix}. \tag{11}$$

Where the four pole parameters of the shaft and the bearing are given by

$\alpha_{11}^1 = \cos(k_r l), \alpha_{12}^1 = z\omega\sin(k_r l), \alpha_{21}^1 = -\sin(k_r l)/z\omega, \alpha_{22}^1 = \cos(k_r l)$ $c_r = \sqrt{E/\rho}$, $k_r = \omega/c_r$ $z = \sqrt{E\rho}A$,

$\alpha_{11}^2 = 1, \alpha_{12}^2 = 0, \alpha_{21}^2 = -1/(K_B + j\omega C_B), \alpha_{22}^2 = 1$.

A part of the gear housing is modeled as a clamped circular plate. The governing equation of the circular plate with viscous damping subject to a force F_{3P} at r_o and θ_o is given by[8]

$$D\nabla^4 w + C_P\frac{\partial w}{\partial t} + \rho h\frac{\partial^2 w}{\partial t^2} = F_{3P}\delta(r - r_o)\delta(\theta - \theta_o)/r, \quad \text{where } D = Eh^3/12(1-\nu^2). \tag{12}$$

Assuming the solution $w(r,\theta,t) = W(r,\theta)e^{j\omega t}$, the spectral form for this equation is obtained. Modal expansion to derive the forced response of the plate is used. By solving the characteristic equation of the undamped circular plate satisfied with the clamped boundary condition, eigenvalues and the corresponding normal mode are calculated. By substituting the normal mode equation into the spectral equation and using the orthogonality of mode, the out-of-plane displacement of the plate is obtained by

$$W(r,\theta,t)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{F_{3P}H_{mn}(\beta_{mn}r_o)H_{mn}(\beta_{mn}r)}{\varepsilon\pi a^2 D\Lambda_{mn}(\beta_{mn}^4-\beta^4)}\cos(m\theta-m\theta_o)e^{j\omega t}, \tag{13}$$

where the mode normalization constant of $\Lambda_{mn}=J_m^2(\beta_{mn}a)$ proposed by Park[6] is used. If m>0, ε=1, and if m=0, ε=2. $H_{mn}(\beta_{mn}r)=J_m(\beta_{mn}r)-\frac{J_m(\beta_{mn}a)}{I_m(\beta_{mn}a)}I_m(\beta_{mn}r)$, $\beta_{mn}{}^4=(\omega_{mn}{}^2\rho h)/D$, $\beta^4=(\omega^2\rho h-i\omega C_p)/D$.

Since the plate and the bearing are joined, the force balance equation and the displacement continuity equation at the joint are given by

$$F_{3B}=F_{3P}. \tag{14}$$

$$U_{3B}=U_{3P}. \tag{15}$$

Here U_{3P} is the displacement of the plate at r_o and θ_o given by

$$U_{3P}=W(r_o,\theta_o)=F_{3P}/K_P, \tag{16}$$

where $1/K_P=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\{H_{mn}(\beta_{mn}r_o)\}^2}{\varepsilon\pi a^2 D\Lambda_{mn}(\beta_{mn}^4-\beta^4)}$.

To obtain natural frequencies of the gear system, the receptance method [9-10] is used. The definition of the receptance method is given by

$$\frac{U_{3B}}{F_{3B}}=\frac{U_{3P}}{F_{3P}}. \tag{17}$$

Substitution of Eqs. (11) and (16) into Eq. (17) and the free vibration condition produce the equation for natural frequencies of the system as follows:

$$\frac{\alpha_{22}}{\alpha_{12}}=\frac{1}{K_P}. \tag{18}$$

2.4 Radiated Noise The clamped damped circular plate excited by a point force located at (r_o,θ_o) is mounted in the infinite baffle and its surface vibration radiate in air. To derive the acoustic pressure due to the plate mode (m,n), the Rayleigh integral [11-12] is applied as follows:

$$p(R,\gamma,\theta',t)=\frac{j\omega\rho_A}{2\pi}e^{j\omega t}\int_S\frac{v_n(r,\theta)e^{-jk_AR'}}{R'}dS(r,\theta), \tag{19}$$

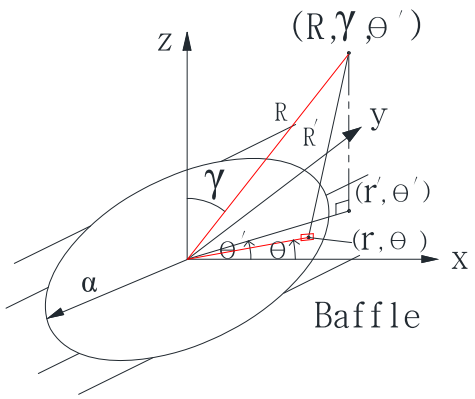

Fig.2 Coordinate system for noise

where (R,γ,θ′) are spherical coordinates and (r,θ) are polar coordinates on the plate surface as shown in Fig. 2. $v_n(r,\theta)$ is the normal velocity amplitude on the plate surface. In the far field of R>>a, R' in the denominator of Eq. (19) may be approximated by R. However, since $\exp(-jk_AR')$ of Eq. (19) is a rapidly varying function of R, the term must be left in the integral[11], and R' in the exponential term may be approximated by[12]

$$R'\cong R-r\sin\gamma\cos(\theta'-\theta). \tag{20}$$

From these approximations, Eq. (19) becomes

$$p(R,\gamma,\theta',t)=\frac{j\omega\rho_A}{2\pi R}e^{j\omega t-jk_AR}\int_0^{2\pi}\int_0^a v_n(r,\theta)\exp(jk_Ar\sin\gamma\cos(\theta'-\theta))rdrd\theta. \tag{21}$$

Differentiation of Eq. (13), the out-of-plane displacement of the plate, with respect to time produces the normal velocity amplitude as follows:

$$v_n(r,\theta) = j\omega \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \frac{F_{3P} H_{mn}(\beta_{mn} r_o) H_{mn}(\beta_{mn} r)}{\varepsilon \pi a^2 D \Lambda_{mn}(\beta_{mn}^4 - \beta^4)} \cos(m\theta - m\theta_o) \cdot \tag{22}$$

Application of Bessel function relations [13] and many mathematical operations give the solution of the following integral:

$$\int_0^{2\pi} \cos(m\theta - m\theta_o)\exp(jk_A r \sin\gamma \cos(\theta' - \theta))d\theta = \cos(m\theta' - m\theta_o) 2\pi j^m J_m(k_A r \sin\gamma) \quad . \tag{23}$$

Therefore, the acoustic pressure due to the plate mode (m,n) in the far field is derived by

$$p(R,\gamma,\theta',t) = \frac{\rho_A \omega^2 F_{3P}}{R} \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \frac{-j^m H_{mn}(\beta_{mn} r_o)}{\varepsilon \pi D \Lambda_{mn}(\beta_{mn}^4 - \beta^4)} \cos(m\theta' - m\theta_o) \Delta e^{j\omega t - jk_A R} \quad , \tag{24}$$

where $\Delta = \int_0^1 H_m(\beta_{mn} r) J_m(k_A r \sin\gamma)(r/a) d(r/a)$.

3.0 Results and Discussion

Table 1 - The data of helical gears and wind turbine.

	Driving gear	Driven gear
Normal module	2.5	
Normal pressure angle [deg]	20°	
Center distance [mm]	150	
Whole depth [mm]	5.6	
Helix angle [deg]	25°	
Number of teeth	89	19
Face width [mm]	48	50
Outside diameter [mm]	250.95	59.10
Pitch diameter [mm]	245.50	52.41
Amount of add. mod.[mm]	0.0893	0.3372
Air density [kg/m^3]	1.21	
Nominal rotor angular velocity (rpm)	15	
Rotor diameter(m)	26	
Rotor swept area (m^2)	530.9	
Average wind speed(m/s)	15	

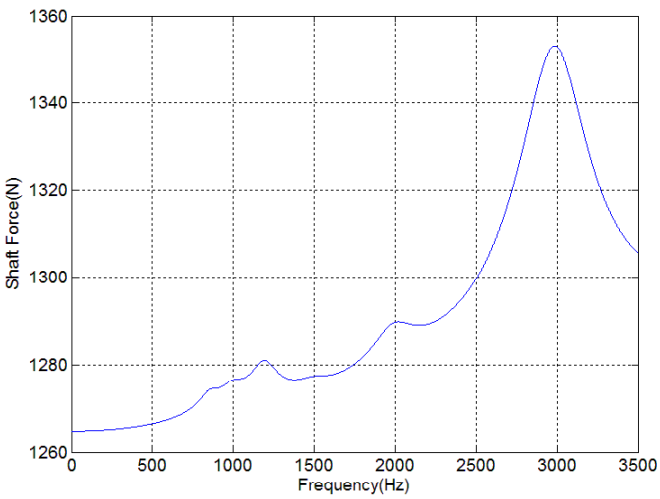

Fig. 3 Shaft input force.

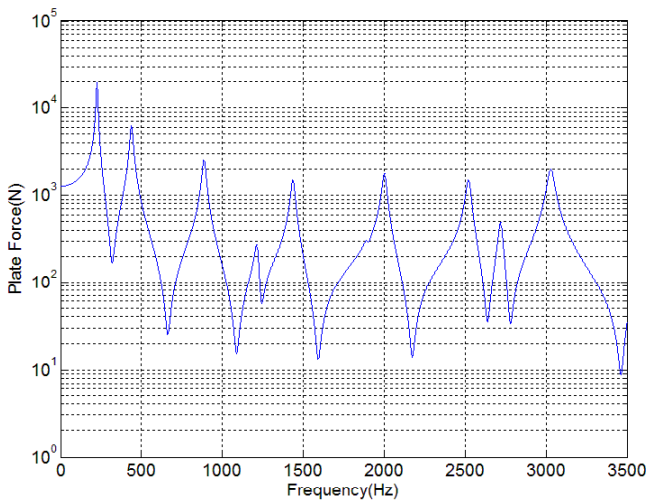

Fig. 4 Plate exciting force.

For the prediction of helical gear noise, the gear and wind turbine data of Table 1 are used. Aerodynamic torque by wind turbine data produces 136.3N·m and the transmitted load of the driving gear is 1196.3 N. The material of gears is steel and gears assume to have the true involute profiles without lead errors and lead crowning. The mesh stiffness of helical gears is obtained by an in-house program[14], which is calculated by the load-deformation equation along the contact. Axial excitation force of the helical gear transmitted to the shaft is obtained by solving the linearized equation of motion in the rotational direction. The damping coefficient of the equation uses 0.07.

The shaft-bearing-plate model uses the same material and size as validation analysis. The steel shaft with the diameter of 20 mm and the length of 265 mm and an aluminum alloy circular plate with the diameter of 400 mm and thickness of 5mm are used. Since bearing stiffness varies very little between static and dynamic conditions, the static stiffness of the bearing is sufficient. The bearing stiffness and damping use 6.84×10^{7} N/m and 760 kg/s respectively. The plate damping of 2,040 kg/m^{2}s and 25 plate modes (m=4, n=5) for the out-of-plane displacement and the sound pressure of the plate are used like validation analysis. All analysis is investigated in the frequency domain within 3 kHz.

Figure 3 is the axial shaft force calculated by Eq. (10). Since the natural frequency of gears in single degree of freedom is 5,987 Hz, the most peak near 3 kHz comes from sub harmonics due to parametric excitation by the fluctuation of mesh stiffness. Figure 4 shows the plate exciting force, which has many peaks related to natural frequencies. Figure 5 is the out-of-plane displacement of the plate calculated at the position (X=100 mm, Y=0), and the displacement also has many peaks related to natural frequencies. Figure 6 is sound pressure level at the observation point of R=2000 mm, γ=45°, θ'=45° radiated in air from the plate. Sound pressure level is expressed as dB re 20 μ Pa. It also has many peaks related to natural frequencies

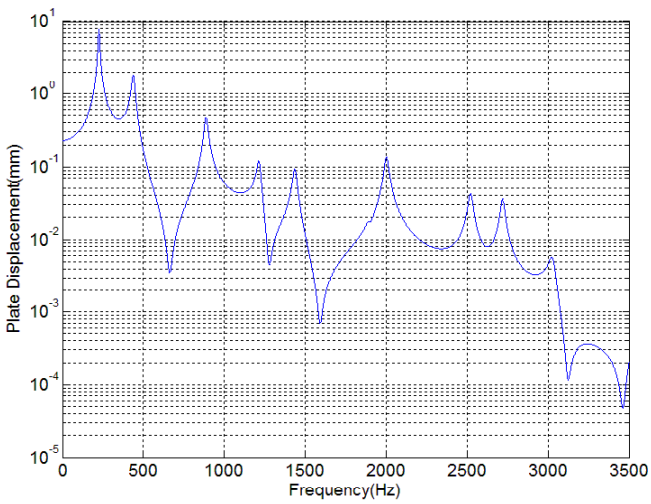

Fig. 5 Plate displacement at (X=100 mm, Y=0).

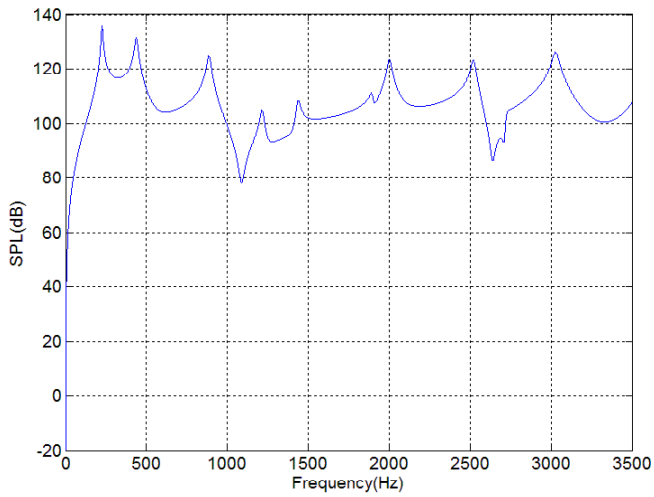

Fig. 6 Sound pressure level at r=2000mm, γ=45°, θ'=45°.

4.0 Conclusion

This work dealt with noise prediction of the simplified gear system with one stage of helical gears as the model problem of wind turbine gearbox. The simplified system was modeled by shaft-bearing-circular plate with helical gears. An exciting torque of the system used the aerodynamic torque of the wind turbine. The torque was transformed into an axial shaft force from single degree of freedom model of helical gears. The four pole parameters for a rod and a spring were used to model the shaft and the bearing. Transfer matrix with four pole parameters for the shaft-bearing-plate model was used to obtain the force and the displacement of the plate by the axial shaft force. The radiated noise from the simplified gear system model of the damped circular plate mounted in the infinite baffle was calculated.

Acknowledgements

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NOMENCLATURE

a	the radius of the circular plate	ξ	damping coefficient
A_w	the area	β_b	base helix angle
C	damping	δ	Kronecker delta
C_p	power coefficient	ρ	density
E	modulus of elasticity	ν	Poisson's ratio
e_j	tooth error	λ	the tip speed ratio
F	force	Ω_m	nominal angular velocity
h	plate thickness	Subscripts	
I_m	modified Bessel functions of the first kind	1	Driving gear
j	complex number	2	Driven gear

J_m	Bessel functions of the first kind	A	air
K	stiffness	B	bearing
k	wave number	G	gear
R	the wind rotor radius	P	plate
R_b	base circle radius	S	shaft
T	Torque		
U	Displacement		
V_m	the average wind speed		